GCE

Mathematics A

H240/02: Pure Mathematics and Statistics

Advanced GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject-specific Marking Instructions for A Level Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.

When a value is not given in the paper accept any answer that agrees with the correct value to **3 s.f.** unless the question specifically asks for another level of accuracy.
NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark is lost for each distinct accuracy error.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			FinalVersion Answer	Mark	AO	Guidance	
			NB Answers must be correct to 3 sf, except where otherwise indicated. If correct answer seen (to ≥ 3 sf), ignore later rounding.				
1						**Ignore incorrect** $\int$ **or dx in all parts**	
1	**(a)**	**(i)**	$\frac{(2x+1)\times 2x - x^2\times 2}{(2x+1)^2}$ oe (eg = $\frac{2x^2+2x}{(2x+1)^2}$ or $\frac{2x(x+1)}{(2x+1)^2}$ oe) Alternative method $x^2(-2)(2x+1)^{-2} + 2x(2x+1)^{-1}$ oe	**B1** **B1** **B1** **[3]**	**1.1a** **1.1** **1.1**	$2x(2x+1)$ <u>or</u> $-2x^2$ oe in numerator B1 Correct denominator B1 Correct numerator B1 No need to see this $\pm 2x^2(2x+1)^{-2}$ oe B1 $+ 2x(2x+1)^{-1}$ oe B1 All correct B1	Condone missing brackets 1st B1 Allow correct equivalent forms ISW for further "simplifications" Allow correct equivalent forms ISW for further "simplifications"
1	**(a)**	**(ii)**	$(2x-3)\sec^2(x^2-3x)$ oe	**B1** **B1** **[2]**	**1.1a** **1.1**	B1 for $\sec^2(x^2-3x)$ B1 for all correct	Condone missing brackets 1st B1 Condone $\sec^2(x^2-3x)(2x-3)$ ISW for further "simplifications"
			Allow without mod in both parts (b) and (c)				
1	**(b)**		$x = (u+1)^2$, $\frac{dx}{du} = 2(u+1)$ oe or $\frac{du}{dx} = 0.5x^{-0.5}$ oe $2\int \frac{(u+1)}{u}du$ or $2\int\left(1+\frac{1}{u}\right)du$ oe $= 2(u + \ln\lvert u\rvert)$ $(+c)$ $= 2(\sqrt{x} - 1 + \ln\lvert\sqrt{x}-1\rvert) + c$ oe or $2(\sqrt{x} + \ln\lvert\sqrt{x}-1\rvert) + c$ oe or $2\sqrt{x} + \ln(\sqrt{x}-1)^2 + c$ oe	**M1** **A1** **A1** **A1** **[4]**	**1.1a** **2.5** **2.1** **1.1** 	<u>EITHER</u> attempt x in terms of u & diff' <u>OR</u> attempt $\frac{du}{dx}$ & obtain $k\,x^{-0.5}$ oe Allow $k\int \frac{(u+1)}{u}du$ or $k\int\left(1+\frac{1}{u}\right)du$ Allow without + c here All correct incl + c	Allow in form dx = ... or du = ... or $\int \frac{(ku+j)}{u}du$ or $\int\left(k+\frac{j}{u}\right)du$ **Not penalise +c in both (b) &(c)** ISW for further "simplifications" Integration by parts: Use same scheme.

1	**(c)**		$\ln\lvert 2x^2-8x-1\rvert$ or $\ln\lvert \frac{1}{2}x^2-2x-\frac{1}{4}\rvert$ seen $\frac{1}{4}\ln\lvert 2x^2-8x-1\rvert+c$ or $\frac{1}{4}\ln\lvert \frac{1}{2}x^2-2x-\frac{1}{4}\rvert+c$	**M1** **A1** **[2]**	**1.2** **1.1**	or $u = 2x^2-8x-1$ and $\ln\lvert u\rvert$ seen All correct including + c Correct answer seen: M1A1 even if eg $(x-2)\frac{\ln\lvert 2x^2-8x-1\rvert}{4x-8} = \frac{1}{4}\ln\lvert 2x^2-8x-1\rvert$	or $u = x-2$ and $\ln\lvert 2u^2-9\rvert$ seen **Not penalise +c in both (b) &(c)** ISW for further "simplifications
2	**(a)**		$= -48384$ or -48400	**B1** **[1]**	**1.1**	Allow $-48384x^5$	
2	**(b)**	**(i)**	$1+0.5\times 3x+\frac{0.5\times(-0.5)}{2}\times(3x)^2$ $+\frac{0.5\times(-0.5)(-1.5)}{3!}\times(3x)^3$ $= 1+\frac{3}{2}x-\frac{9}{8}x^2+\frac{27}{16}x^3$ or $1+1.5x-1.125x^2+1.6875x^3$	**M1** **A1** **A1** **[3]**	**1.1a** **1.1** **1.1**	M1 for at least 3 terms correct Condone any missing brackets A1 for 3 terms correct A1 for all correct	SC $1+\frac{3}{2}x-\frac{3}{8}x^2+\frac{3}{16}x^3$: M1
2	**(b)**	**(ii)**	$-\frac{1}{3} < x < \frac{1}{3}$	**B1** **[1]**	**1.2**	Allow $\lvert x\rvert < \frac{1}{3}$	
2	**(b)**	**(iii)**	Sub $x = 0.01$ in their expansion gives $\sqrt{1.03} = 1.014889\ldots$ From series $\sqrt{103} = 10.14889(188\ldots)$ From calculator $\sqrt{103} = 10.14889(157\ldots)$ (Hence expansion may be correct)	**M1** **A1** **A1** **[3]**	**3.1a** **1.1** **2.2b**	Allow 1.01489 here (5 dps for series) If no working seen, 10.1488919 or better must be seen as evidence that series has been used. Both these must be seen for A1 Allow without statement	Other correct methods may be seen, eg subst $x = 0.2$ & $\sqrt{1.6}$ 5dps for $\sqrt{103}$ in both

3	(a)	(i)	$(x-3)^2+(y+4)^2=4\cos^2\theta+4\sin^2\theta$ $\Rightarrow (x-3)^2+(y+4)^2=4$ oe ISW	M1 A1 [2]	3.1a 2.1	or $\left(\frac{x-3}{2}\right)^2+\left(\frac{y+4}{2}\right)^2=\cos^2\theta+\sin^2\theta$ or $\left(\frac{x-3}{2}\right)^2+\left(\frac{y+4}{2}\right)^2=1$ oe or $\cos^{-1}\left(\frac{x-3}{2}\right)=\sin^{-1}\left(\frac{y+4}{2}\right)$ M1A1 **ISW for all answers**	Condone sign errors or one arith slip or missing brackets for M1 or $y=-4+2\sqrt{1-(\frac{x-3}{2})^2}$ M1A1 or similar with $x=$ or $y=-4+2\sin(\cos^{-1}\frac{x-3}{2})$ M1A1 or similar with $x=$
3	(a)	(ii)	Centre (3, –4), radius 2	B1f [1]	2.2a	ft their (i) if both consistent with (i) But if absolutely correct, not ft: B1.	
3	(b)		**DR** **NB Allow decimals to 2 sf instead of surds thoughout, except answer to 3 sf** $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$ $=-\frac{1}{2}\cot t$ or $-\frac{1}{2}\frac{\cos t}{\sin t}$ $t=\frac{\pi}{6}$: $\frac{dy}{dx}=-\frac{1}{2}\cot\frac{\pi}{6}$ oe or $-\frac{\sqrt{3}}{2}$ oe **Alternative methods for gradient** $\left(\frac{x}{4}\right)^2+\left(\frac{y}{2}\right)^2=1,\quad \frac{x}{8}+\frac{y}{2}\frac{dy}{dx}=0$ M1 $\frac{dy}{dx}=-\frac{x}{4y}$ A1 $t=\frac{\pi}{6}$: $\frac{dy}{dx}=-\frac{4\cos(\frac{\pi}{6})}{8\sin(\frac{\pi}{6})}$ or $-\frac{1}{2}\cot\frac{\pi}{6}$ or $-\frac{\sqrt{3}}{2}$ M1	M1 A1 M1	3.1a 1.1 1.1	Attempt diff x & y wrt t & find $\frac{dy}{dt}\div\frac{dx}{dt}$ soi Substitute $t=\frac{\pi}{6}$ in their $\frac{dy}{dx}$ Attempt cartesian equn & differentiation soi Substitute $t=\frac{\pi}{6}$ in x (and y) & their $\frac{dy}{dx}$	Allow sign error $\frac{d}{dx}(0.5(16-x^2)^{-0.5})$ or $\frac{dy}{dx}=\frac{1}{4}(16-x^2)^{-0.5}(-2x)$ oe

			Equn of L is $y - 2\sin\frac{\pi}{6} = -\frac{\sqrt{3}}{2}(x - 4\cos\frac{\pi}{6})$ oe or $y - 1 = -\frac{\sqrt{3}}{2}(x - 2\sqrt{3})$ oe $0 - 1 = -\frac{\sqrt{3}}{2}x + 3$ oe Cuts at $(\frac{8\sqrt{3}}{3}, 0)$ oe or (4.62, 0) (3 sf)	**M1** **M1** **A1** **[6]**	**1.1** **1.1** **2.2a**	or $y = -\frac{\sqrt{3}}{2}x + c$ **&** subst $(4\cos\frac{\pi}{6}, 2\sin\frac{\pi}{6})$ or $y = -\frac{\sqrt{3}}{2}x + 4$ oe or $0 = -\frac{\sqrt{3}}{2}x + 4$ oe Allow just $\frac{8\sqrt{3}}{3}$ or 4.62 (3 sf)	ft their grad (not -ve reciprocal) Must not involve t This mark may be implied by next mark Subst $y = 0$ in their line equn, not involving t Allow equivalents, eg $\frac{8}{\sqrt{3}}$
4			**In all parts ignore nos except 20, & 1020**			**BOD if describe growth rather than rate in (a) and (b)**	Condone muddle between P and growth of P in (a) and (b)
4	**(a)**		A: Growth (rate) increases, then decreases Grows slowly, then quickly, then slowly B: Growth (rate) decreases Grows quickly then slowly Both	**B1** **[1]**	**2.2b**	Allow increase, constant, then decrease or "levels off", "tails off", "plateaus" Allow "levels off", "tails off", "plateaus"	NOT "P" decreases, for A or B Ignore "exponentially"
4	**(b)**	**(i)**	A: P (decreases and) tends to 20 or (Decreases and) doesn't go below 20	**B1** **[1]**	**3.4**	Allow (Decrease and) reach 20, **Must mention 20** (as population, not years)	Ignore all else
4	**(b)**	**(ii)**	B: P tends to 1020 oe P doesn't exceed 1020	**B1** **[1]**	**3.4**	Growth is asymptotic around 1020 Settles at 1020. Saturates at 1020 Converges to 1020. Allow reaches 1020 Plateaus at 1020. Asymptote at 1020 **Must mention 1020**	NOT: Pop increases, but slowly Diverges to 1020 Tends to 1020, then down Ignore all else
4	**(c)**	**(i)**	A: Food (almost) runs out, or is used up oe or becomes very low or there will be a shortage oe or begins to run out	**B1** **[1]**	**3.5a**	or will only support a population of 20 Won't sustain large nos. Insufficient NB "Limited" allowed in c(ii), not c(i)	NOT: just Limited, Finite NOT: just "Decreases" Ignore all else
4	**(c)**	**(ii)**	B: Food sufficient to support a pop ≈ 1020 Enough to sustain equilibrium (or population) Barely enough, can't support increase in P Food limited so pop can't continue to grow	**B1** **[1]**	**3.5a**	Stays stable Sustainable Constant	Must imply at least two of: 1. Food won't run out and 2. Food limited or equilibrium 3. Can't support increase in P Ignore all else

5	**(a)**		An example of a correct method:			Other correct methods may be seen eg $lr = 4$: B1, find h ito l & r: B1, Subst h & lr into V: M1, convincing: A1	**NB** Must be clearly using this alternative method to score any marks on this scheme.
			$l = \frac{4}{r}$ or $l = \frac{4\pi}{r\pi}$ exactly (not $lr = 4$) $(h = \sqrt{l^2 - r^2})$	**B1**	**3.1a**	Express l correctly in terms of r May be implied	Allow $l = \frac{4-r^2}{r}$ oe B1 $(h^2 = l^2 - r^2)$
			$h = \sqrt{\frac{16}{r^2} - r^2}$ or $\frac{\sqrt{16-r^4}}{r}$ oe	**B1**	**1.1**	Express h (or h^2) correctly in terms of r alone	or $h^2 = \frac{16}{r^2} - r^2$ or $\frac{16-r^4}{r^2}$
			$V = \frac{1}{3}\pi r^2\sqrt{\frac{16}{r^2} - r^2}$ or $\frac{1}{3}\pi r^2\frac{\sqrt{16-r^4}}{r}$ oe	**M1**	**1.1**	Sub their h (in terms of r alone) into $\frac{1}{3}\pi r^2 h$	
			$(= \frac{\pi}{3}\sqrt{16r^2 - r^6}$ **AG)**	**A1** **[4]**	**2.1**	Must see a correct previous expression in terms of r only, and the answer	
5	**(b)**	**DR**	$\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{\pi}{3}\sqrt{16r^2 - r^6}\right)$	**M1**	**1.1a**	Attempt differentiate V or $\frac{V}{\pi}$ or $3V$	or $\frac{3V}{\pi}$ or $\sqrt{16r^2 - r^6}$ or $16r^2 - r^6$
			$\frac{\pi(32r - 6r^5)}{3\times 2\sqrt{16r^2 - r^6}} = 0$ oe (Their derivative = 0)	**A1**	**2.1**	Correct derivative of one of the above Condone missing brackets.	All subsequent marks can be scored even if this A1 not scored.
			$r = \frac{2}{\sqrt[4]{3}}$ or $\sqrt[4]{\frac{16}{3}}$ oe or 1.52 (3 sf) Allow 1.5 or $r^2 = \frac{4}{\sqrt{3}}$	**A1**	**1.1**	Lose this mark if incorrect values of r also given, eg $r = \pm 2$ obtained from $(16r^2 - r^6)^{-\frac{1}{2}} = 0$	Allow without $r = 0$
			$r = -\frac{2}{\sqrt[4]{3}}$ or –1.52 invalid OR $r = 0$ invalid or $r > 0$ $(V_{\max} = \frac{\pi}{3}\sqrt{16\times 1.51967^2 - 1.51967^6}$)	**B1f**	**3.2a**	Comment needed, about their negative r (ft) or about $r = 0$	T & I: 5.20 (3sf) SC B2 5.2 (2 sf) SC B1
			Max $V = 5.20$ (3 sf) Allow 5.2 or a.r.t. 5.2	**A1** **[5]**	**1.1**	Condone $V = 5.20$ m^3	

6			$2^{2k} - 1$ or $4^k - 1$ (where k is an integer > 1) $= (2^k)^2 - 1$ $= (2^k - 1)(2^k + 1)$ $(2^k + 1) > 1$ and $k > 1$, hence $(2^k - 1) > 1$ Hence $(2^k - 1)(2^k + 1)$ is the product of two integers, both > 1, and hence $2^n - 1$ is not prime	**M1** **A1** **M1** **A1** **[4]**	**3.1a** **2.1** **1.1** **2.2a**	or $2^{2k+2}-1$ or $2^{2k+4}-1$ Allow $2^{2n}-1$ $= (2^{k+1}-1)(2^{k+1}+1)$ oe or $(2^{k+2}-1)(2^{k+2}+1)$ oe Both statements needed	**Induction:** Assume $2^k - 1$ is ÷ by 3 (k even) M1 Let $2^k - 1 = 3p$ (p integer) $2^{k+2} - 1 = 4\times2^k - 1$ M1 $= 4\times2^k - 4 + 3$ $= 4(2^k - 1) + 3$ $= 4 \times 3p + 3$ A1 which is ÷ by 3 When $k = 2$: $2^2 - 1 = 3$ so ÷ by 3 A1 Hence true for all even n. Claim true
7			**DR** $u^2 = 36x^4 + 12x^3 + x^2$ So $36x^4 + 12x^3 + 7x^2 + x - 2 = u^2 + 6x^2 + x - 2$ Equn reduces to $u^2 + u - 2 = 0$ $u = -2$ or 1 $6x^2 + x = -2$ has no roots because $\Delta = 1 - 48 < 0$ $6x^2 + x = 1$ has roots $x = \frac{1}{3}$ or $-\frac{1}{2}$	**M1** **A1** **A1** **B1** **A1** **[5]**	**3.1a** **2.1** **1.1** **3.2a** **1.1**	$(36x^4 + 12x^3 + 7x^2 + x - 2) \div (6x^2 + x)$ $= 6x^2 + x + 1$ rem -2 $((6x^2 + x)(6x^2 + x + 1) = 2)$ $u(u + 1) = 2$ **BC** Must see correct calc'n for Δ and "< 0" for their quadratic equation **BC** Ignore any answers from $u = -2$ SC If M1 gained but incorrect or inadequate method & correct answers: M1A0A0B0A1	M1 for attempt $(6x^2 + x)^2$ or attempt ÷ LHS by $(6x^2 + x)$, at least 2 terms correct or obtain any correct equn in terms of x and u or $x = \frac{-1\pm\sqrt{47}i}{12}$ given instead of "no roots" etc Otherwise correct ans without any correct working: no marks. (Because **DR**)
8	**(a)**		65 Quartiles are 76 and 61 = 15	**B1** **M1** **A1** **[3]**	**1.1** **1.1** **1.1**	 Allow (75 to 77) and (59 to 63) Must come from 76 – 61 SC Misread 5 | 6 = 5.6: lose B1 only	
8	**(b)**		Mean = 69 sd = 10.5 (3 sf)	**B1** **B1**	**1.1** **1.1**	Allow 6.9 Allow 1.05 (3 sf)	

				[2]			
8	**(c)**		Less (or not) affected by the outlier or anomaly 99 Mean (more) affected by the outlier of 99	**B1** **[1]**	**1.1**	oe, but must mention 99 Allow "Median is less skewed by the 99"	Ignore all else, eg "more accurate"
9	**(a)**	**(i)**	0.761 or 0.762 (3 sf)	**B1** **[1]**	**1.1**	**BC** Allow 0.76	
9	**(a)**	**(ii)**	62.0 (3 sf)	**B1** **[1]**	**1.1**	**BC** Allow 62 or 61.9	Allow $m \geq 62.0$
9	**(a)**	**(iii)**	Use of $\bar{X}$ eg " $\bar{X}$ " or "mean" or $\frac{18}{10}$ or $\sqrt{\frac{18}{10}}$ $\bar{X} \sim N(55, \frac{18}{10})$ $P(\bar{X} < \frac{530}{10})$ **dep** $\sigma^2 = \frac{18}{10}$ = 0.0680 (3 sf)	**M1** **M1** **M1** **A1** **[4]**	**1.1a** **3.3** **3.4** **1.1**	$\mu = 550$ seen or implied $\Sigma X \sim N(550, 180)$ Correct $P(\Sigma X < 530)$ **dep** $\sigma^2 = 180$ = 0.0680 (3 sf) **Allow 0.068**	 May be implied Stated or implied Correct answer from limited (or no) working: M1M1M1A1
9	**(b)**		$P(Y < 72) = 0.75$ | $P(Y < 62) = 0.25$ $\Phi^{-1}(0.75)$ or 0.674 | $\Phi^{-1}(0.25)$ or –0.674 $\frac{72-67}{\sigma}$ | $\frac{62-67}{\sigma}$ $\frac{72-67}{\sigma} = 0.674$ | $\frac{62-67}{\sigma} = -0.674$ σ = 7.41 or 7.42 (3 sf) **Trial and Improvement** $\Phi^{-1}(0.75)$ or 0.674 or $\Phi^{-1}(0.25)$ or –0.674 eg $\sigma = 8$: $67 - 8\times0.674 = 61.6$ $\sigma = 7$: $67 - 7\times0.674 = 62.3$ $\sigma = 7.41$: $67 - 7.41\times0.674 = 62.0 \Rightarrow \sigma = 7.41$ or $\sigma = 7.42$: $67 - 7.42\times0.674 = 62.0 \Rightarrow \sigma = 7.42$	**M1** **M1** **M1** **A1** **A1** **M2** **M1** **A1** **A1**	**3.1b** **2.4** **2.1** **1.1** **1.1** 	oe May be implied, eg on diagram ±0.674 implies M1M1 **Allow 0.67** oe, eg $5 = 0.674\sigma$ A1 for correct equn, allow 0.67 **SC correct answer with no working or irrelevant working: SC B3** (because "determine" rather than "find") May be implied At least one correct trial Trials leading to values either side of 62 Correct trial using $\sigma = 7.41$ or 7.42 and conclusion $\sigma = 7.41$ or 7.42	NB $P(62<Y<72) = 0.5$ no mks yet M1M1M1 may be implied by A1 Must be seen or SC B2 if correct to 2 sf

				[5]			
10			**See the exemplars at the end of the MS** **Allow 2 sf throughout**				**If 1-tail test:**
			H_0: $\mu = 0.034$ H_1: $\mu \neq 0.034$ where μ = (pop) mean pollutant level Allow any letter (except X and $\bar{X}$: B0B0)	**B1** **B1**	**1.1** **2.5**	Subtract B1 for each error eg: 1-tail B1B0 undefined μ B1B0 not in terms of parameter B1B0 μ = sample mean implied B1B0 Not include value 0.034 B0B0 eg $H_0 = 0.034$ etc: B0B0	H_0: $\mu = 0.034$, defined μ, B1 H_1: $\mu < 0.034$ B0
			N(0.034, $\frac{0.0000409}{50}$)& $\bar{X}$ <0.0325 (condone >, =) or $\frac{a-0.034}{0.00640 \div \sqrt{50}} = -1.96$	**M1**	**3.3**	Stated or implied eg by 0.0486 or 0.951 or 0.322 (2 sf) even if within incorrect statement eg P(X = 0.0325) = 0.0486	As LH column M1
			P($\bar{X}$ < 0.0325) = 0.0486 or CV is 0.0322 or acceptance region is 0.0322 to 0.0358 or P($\bar{X}$ > 0.0325) = 0.951	**A1***	**3.4**	**BC** or $\frac{0.0325-0.034}{0.00640 \div \sqrt{50}} = -1.66$	P($\bar{X}$ < 0.0325) = 0.0486 or CV is 0.0325(1) A1
			0.0486 > 0.025 or 0.0325 > 0.0322 or 0.0325 is in AR or 0.951 < 0.975 A1A1	**A1**	**1.1**	or 1.66 < 1.96 or –1.66 > –1.96 dep A1* Must be seen, allow on diag	Comp 0.05 or ±1.645 A1 or 0.0325 < 0.0325(1) or 0.0325 is within CR No more marks
			Do not reject H_0 Or Insufficient (or No) evidence to reject H_0 Allow Accept H_0	**M1**	**1.1**	Dep 0.0486 or 0.0322 or –1.66 or 0.951 < 0.975 seen, or P($\bar{X}$ < 0.0325) stated or implied (possibly with wrong prob leading to opposite conclusion**) but 0 if:951 > 0.025 M0	May be implied by conclusion Condone Reject H_1
			Insufficient (or No) evidence that (mean) pollutant level has changed oe or eg "It is unlikely that level has changed"	**A1f**	**2.2b**	In context. Context may be implied by eg "level" or "pollutant"	Not "There is evidence that mean level has not changed" A0 Not definite, eg

			"We can assume that level hasn't changed"			ft only ** above	"Mean level has not changed "A0
				[7]			
11	**(a)**		$k > 1.4$ (allow $k > 1.1$ to 1.6) $k < 0.25$ (allow $k < 0.2$ to 0.3) or $1.4 < k <$ ..... B1 ..... $< k < 0.25$ (ranges as above) B1	**B1** **B1** **[2]**	**2.2b** **2.2b**	Allow $\geq$ and $\leq$ SC: $0.25 < k < 1.4$: B1B0 (ranges as on left)	Allow "x"
11	**(b)**	**(i)**	$0.797 > 0.5577$ or $-0.797 < -0.5577$ or $\lvert-0.797\rvert > 0.5577$	**B2** **[2]**	**3.1b** **3.2a**	$0.797 > 0.6055$ or $-0.797 < -0.6055$ B1 ± 0.5577 B1	Allow $\geq$ or $\leq$
11	**(b)**	**(ii)**	There are clusters (or groups etc.) Apparent good correlation caused by clusters or Two clusters with no –ve corr'n within them or a comment similar to one of the above. AND Conclusion unreliable or Value of r is misleading oe	**B1*** **B1** dep B1* **[2]**	**2.3** **3.5b**	 or Not bivariate normal distribution B1 so use of tables for r not valid B1	NOT Too scattered Not represent whole pop Small sample Clusters not on reg line B1B0
11	**(c)**		High prop of 65+ or Low prop of 18-24 Prop of young very similar, or ≈ 0.06 Proportion of senior to young is high	**B1** **[1]**	**2.2b**	If consider only <u>one</u> age-group, must be proportion not number If consider <u>both</u> age-groups, allow eg Higher number of seniors than young or Many seniors, few young	NOT: Similar proportions of 65+ Population is elderly
11	**(d)**		Top left points contain high prop of 18-24s. (So these LAs may be areas where there are universities or where they can recruit)	 **B1** **[1]**	 **2.2b**	Shows places where large nos of 18-24s Shows where to focus recruiting. So universities can recruit. 18-24s are their target group. No need to specify "Top left group"	Allow "students" or "young" instead of "18-24s" Any implication that diagram enables you to see information about location of young people
12	**(a)**		$k(1 + 2 + 3 + 4 + 5)$ $(= 1)$ $k = \frac{1}{15}$	**M1** **A1**	**3.3** **1.1**	Allow $15k$ $(= 1)$ May be implied	

			P($X = 3$) = $3 \times \frac{1}{15}$ or $\frac{3}{15}$ (= 0.2 **AG**)	**A1** **[3]**	**2.1**	Must see $3 \times \frac{1}{15}$ or $\frac{3}{15}$ and answer 0.2		
12	**(b)**		1 2 3 4 5 $\frac{1}{15}$ $\frac{2}{15}$ $\frac{3}{15}$ $\frac{4}{15}$ $\frac{5}{15}$ oe 0.07, 0.13, 0.2, 0.27, 0.33	**M1** **A1** **[2]**	**1.2** **1.1**	M1 for $\geq$ 3 probs correct, ft their k cao. Allow decimals **(2 dp)** SC: Table with all five probs = 0.2: M1	Allow $X = 0$ or $X = 6$ or $X = 6+$ if prob shown as 0	
12	**(c)**		Both parts. Allow mixture of methods					
12	**(c)**	**(i)**	$\frac{3}{15} \times \frac{4}{15} + \frac{2}{15} \times \frac{5}{15}$ oe $\times 2$ $= \frac{44}{225}$ or 0.196 (3 sf)	**M1** **M1** **A1** **[3]**	**3.4** **3.1a** **1.1**	Correct products added, ft their table 2×(Sum of two products of probs) cao	**Special cases** **2-way table** Count 4 pairs M1 But if (b) correct: M0 ÷ 25 M1 = 0.16 A0	**All probs = 0.2** $0.2^2+0.2^2$ M1 But if (b) correct: M0 $2\times(0.2^2+0.2^2)$ Allow without 2× M1 = 0.16 A0
12	**(c)**	**(ii)**	P(one value is 2 & $T = 7$) = $2\times\frac{2}{15}\times\frac{5}{15}$ $=\frac{4}{45}$ $\frac{P(\text{one value is 2 \& } T=7)}{P(T=7)}$ ($\frac{\frac{4}{45}}{\frac{44}{225}}$ or = $\frac{0.0889}{0.196}$) $= \frac{5}{11}$ or 0.455 (3 sf)	**M1** **A1f** **M1** **A1** **[4]**	**3.4** **1.1** **2.1** **1.1**	Allow without "2×", ft their table ft their table (except if all probs = 0.2) $\frac{\text{Allow any probability}}{\text{Their (c)(i) or their } P(T=7)}$ cao NB not 0.454 Eg: If (i) $\frac{22}{225}$, $\frac{2}{45} \div \frac{22}{225} = \frac{5}{11}$ M1A0M1A0	Count 2 pairs M1 A0 ÷ 4 M1 = 0.5 A0	2×0.2^2 M1 = 0.08 A0 $\frac{0.08}{0.16}$ M1 = 0.5 A0

13			N(5000×0.26, 5000×0.26×0.74) may be implied = N(1300, 962) 1300–2×√"962" or 1300 – 1.96×√"962" (= 1238) (= 1239) P(X < 1239) or P($X \leq$ 1238) or P(X < 1240) or P($X \leq$ 1239) P($X \leq$ 1238) = 0.0233 OR P($X \leq$ 1239) = 0.0251 P($X \leq$ 1238) = 0.0233 AND P($X \leq$ 1239) = 0.0251 AND Largest n is 1239 or $n \leq$ 1239 Example of incorrect method: 1300 – 2×√"962" (= 1238) P(X < 1239) = 0.0251 P(X < 1238) = 0.0233 Largest n is 1239 SC 4	**M1** **M1** **M1** **A1** **A1** **[5]**	**3.1a** **3.3** **3.4** **1.1** **2.2a**	or ϕ^{-1}(0.025) May be implied or –1.96 or 1239(.2) seen One of these attempted, by Binomial or Normal **BC** Correct P($X \leq$ 1238) &P($X \leq$ 1239) seen and conclusion. Example of inadequate method: 1300 –1.96×√"962" (= 1239) or inv Bin(0.025) = 1239 Largest n is 1239 SC 4	or, using Binomial not Normal, B(5000, 0.26) soi M1 Attempt P(X < n) for 1230$\leq n \leq$ 1250 or inverse Bin (0.025) (= 1239) M1 **Allow 0.0232 instead of 0.0233** If use normal to find these probs (0.0265 & 0.0246) A0A0 **NB If two methods used, mark the better one.** **NB:** **SC 1239, no working or incorrect or inadequate working,** **SC: 4 marks out of 5**

Exemplars for Q10

Hypotheses

A H_0: $\mu = 0.034$
H_1: $\mu \neq 0.034$ where μ = (pop) mean pollutant level B1B1

B H_0: $\mu = 0.034$
H_1: $\mu \neq 0.034$ B1B0

C H_0: The (pop) mean pollutant level is 0.034
H_1: The (pop) mean pollutant level is not 0.034 B1B0
See Specimen paper q10 MS "Must be in terms of parameter values"

D $H_0 = 0.034$
$H_0 \neq 0.034$ B0B0

E H_0: $\mu = 0.034$
H_1: $\mu = 0.0325$ where μ = (pop) mean pollutant level B1B0

Probability and conclusion

F No statement of distribution
$P(\bar{X} = 0.0325) = 0.0486$ M1A1
$0.0486 > 0.025$ A1
Don't reject H_0 M1
Likely that mean level of pollutant hasn't changed A1

G $P(\bar{X} = 0.0325) = 0.0486$ M1A1
$0.0486 > 0.025$ A1
Accept H_0 M1
There is evidence that mean level of poll'nt hasn't changed A0

H $P(\bar{X} < 0.0325) = 0.951$ M1A0
$0.951 > 0.025$ A0
Insufficient evidence that poll't level has changed M0A0

I $P(\bar{X} > 0.0325) = 0.951$ M1A1
$0.951 > 0.025$ A0
Sufficient evidence that mean poll't level has changed M0A0

J $\bar{X} \sim N(0.034, 0.000000818)$
$P(\bar{X} < 0.0325) = 0.013$ M1A0
$0.013 < 0.025$ A0
Sufficient evidence that level has changed M1A1

K $\mu \pm 1.96\sigma = 0.0322$ to 0.0358 BOD M1A1
0.0325 lies within this range A1
Reject H_1 M1
Insufficient evidence that level of poll't has decreased A0

L CV = 0.0322 M1A1
0.0325 > 0.0322 A1
Reject H_0. Evidence that level of poll't has changed. M0A0

M $(0.0322 - 0.034) \div \sqrt{0.0000409 / 50} = -1.66$ M1A1
1.66 < 1.96 A1
Don't reject H_0. Level of poll't hasn't changed. M1A0

1-tail

N H_0: $\mu = 0.034$
H_1: $\mu < 0.034$ where μ = (pop) mean pollutant level B1B0

O H_0: $\mu = 0.034$
H_1: $\mu < 0.034$ B0B0

P H_0: The (pop) mean pollutant level is 0.034
H_1: The (pop) mean pollutant level is less than 0.034 B0B0

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